Date ____________________________

In Class Activity

Central Limit Theorem – Sample Means

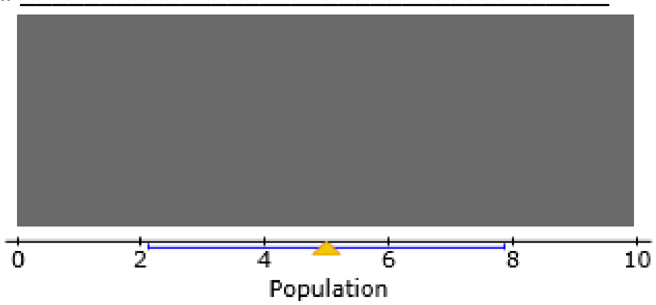
Group Member Names: ____

Part One – Review of Types of Distributions

Consider the three graphs below. Match the histograms with the distribution description.
Write the description of the distribution above each distribution in the blank beside each letter.

Distribution Descriptions: *Normal*, *Uniform*, and *Skewed*

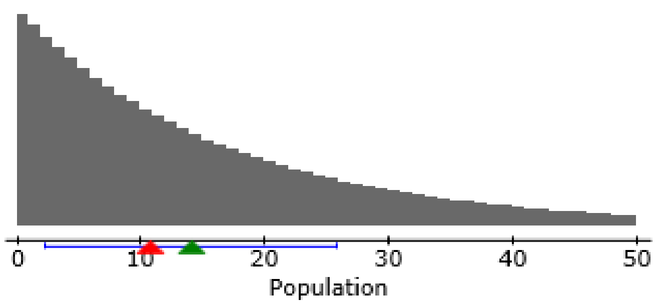
A. ____



Population

Mean	5
Median	5
Std. dev.	2.8862

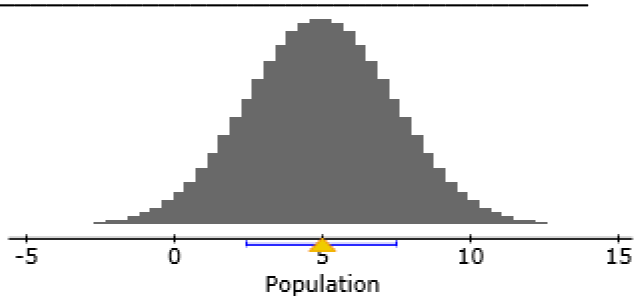
B. ____



Population

Mean	14.0519
Median	10.7484
Std. dev.	11.8255

C.____

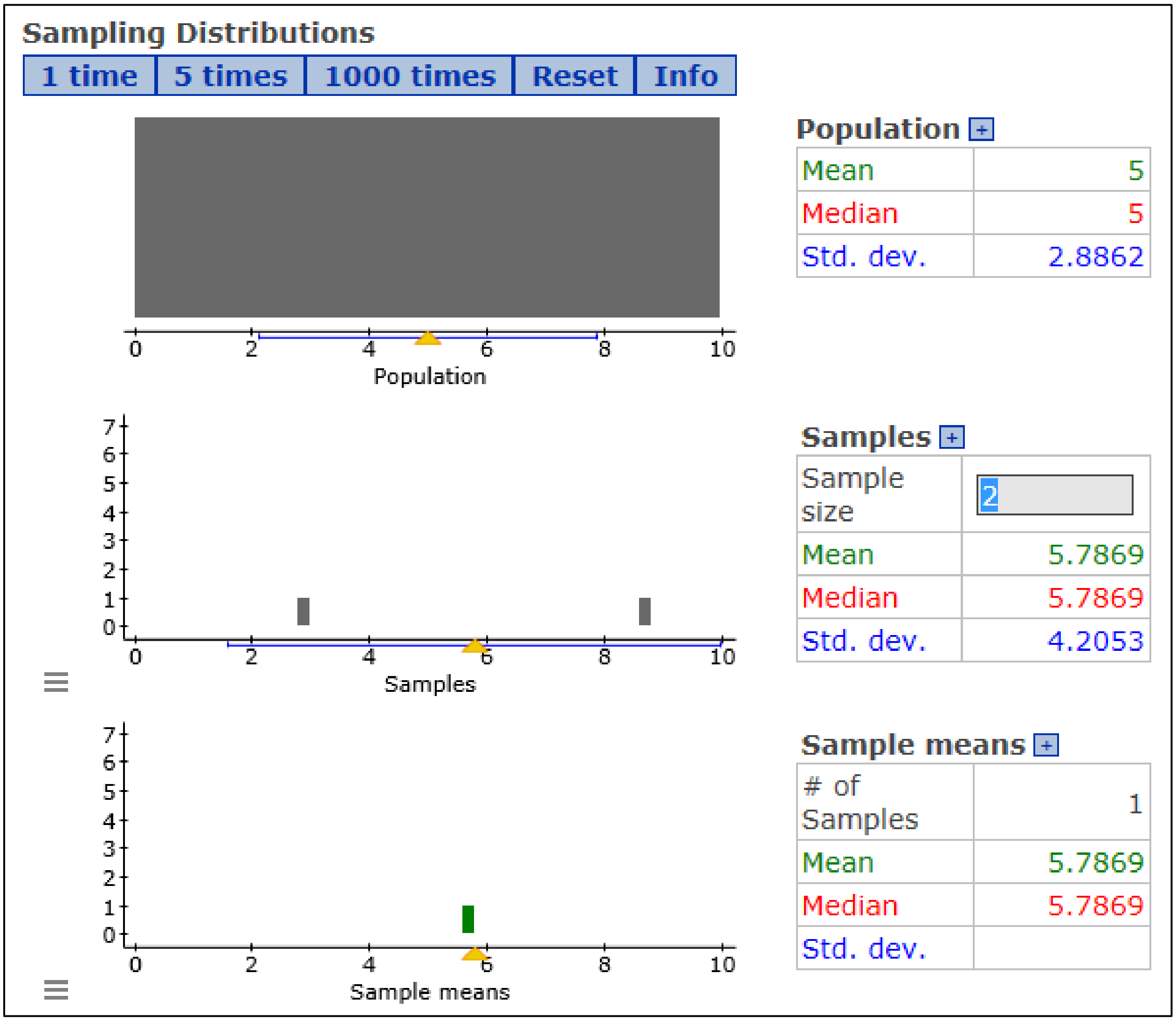


Population

Mean	5
Median	5
Std. dev.	2.5

Part Two – Sampling Distributions from a Uniform Population (n = 2)

1. This image has 3 separate graphs and their corresponding information to the right.



Use this image to answer the following questions.

a. **Graph 1** (top graph) shows the original distribution. What is this distribution called?

b. **Graph 2** (middle graph) contains a sample.

   1) What size is the sample? ________________

   2) What are the approximate numerical values of the sample? ______________________

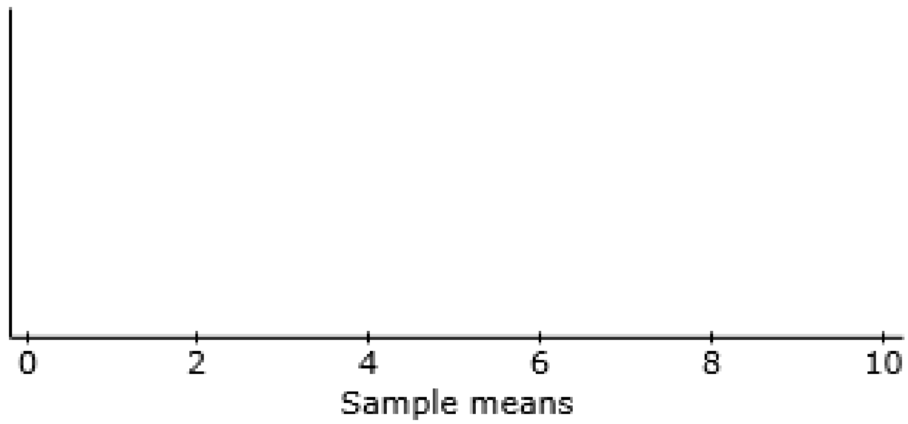
   3) What is the average of the sample? _________________

c. **Graph 3** (bottom graph) contains the sample mean from the sample(s) drawn in Graph 2.

   What is the value of the mean of the last sample? __________________

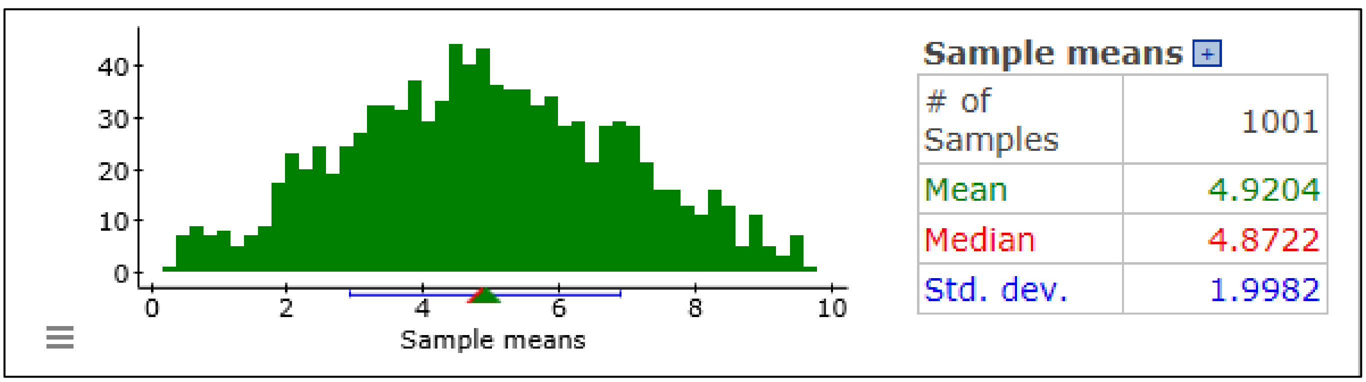
d. If you drew another sample of the same size (n = 2) from the original distribution (top graph) what would be a reasonable estimate for the average of those data points? ____________

e. If you drew 1000 samples of that same size (n = 2) as in the image above, what is your best guess of what the histogram of those 1000 averages would look like? Sketch it below.



2.

a. Compare your estimate of a graph of 1000 averages (from number 1, part e) to the graph below (which has 1001 sample means.) How was your guess?

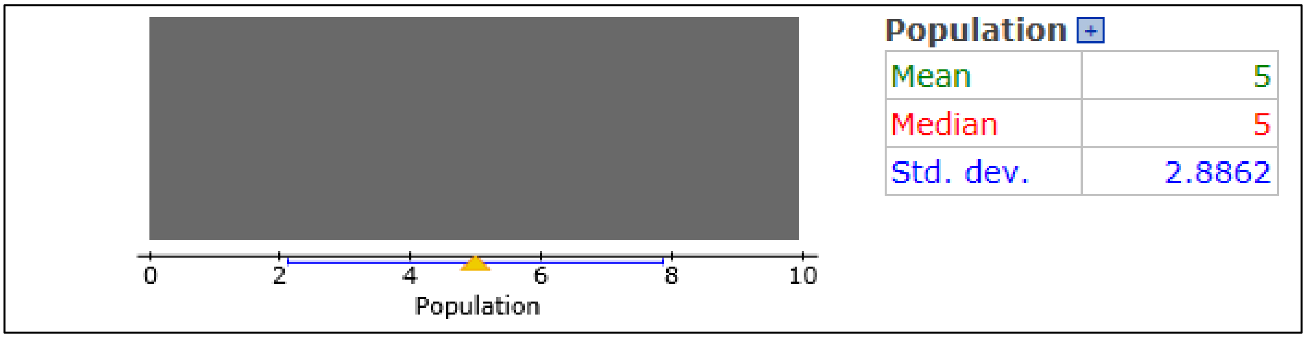


b. What does this histogram represent?

c. What is the basic shape of the histogram? ____

d. What is the mean? __________________

e. Let's compare the original uniform distribution (below, from number 1) with the corresponding sampling distribution of sample means above (from number 2, part a.).



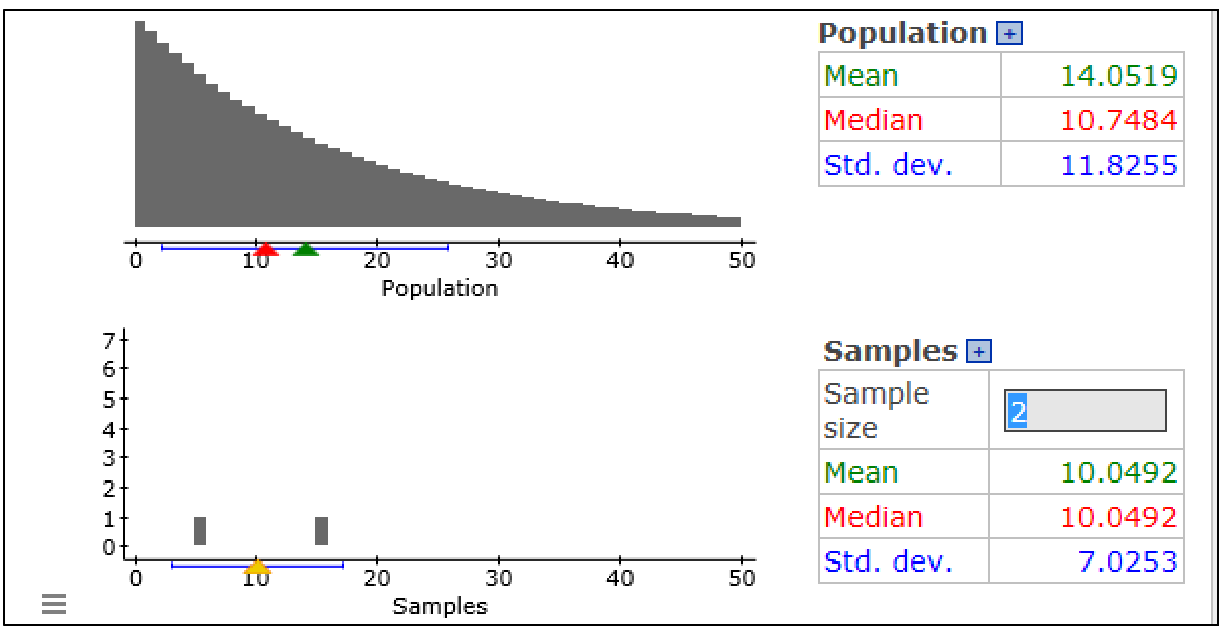
1) How do the **basic shapes** compare?

2) How do the **means** compare?

3. Do you think it a coincidence that the distribution of sample means (from number 2, part a) looks roughly triangular?

Part Three – Sampling Distributions from a Skewed Population (*n* = 2)

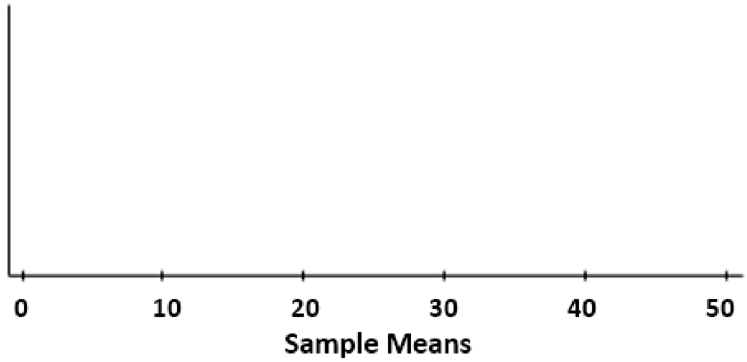
1. Let's start with a different original distribution and stick with a sample size of two.



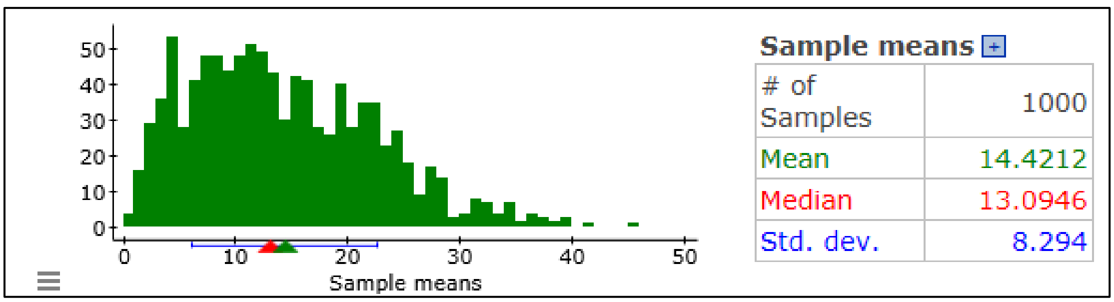
a. For the population, what is the basic shape? ____________________The mean? _____________

b. What size is the sample? _________________ What is the mean of that sample? _____________

c. Sketch your guess of a histogram of a distribution of 1000 sample means from that population.



2. For a sample of size two, here is histogram of 1000 sample means from the skewed distribution in the previous problem. How close was your guess?

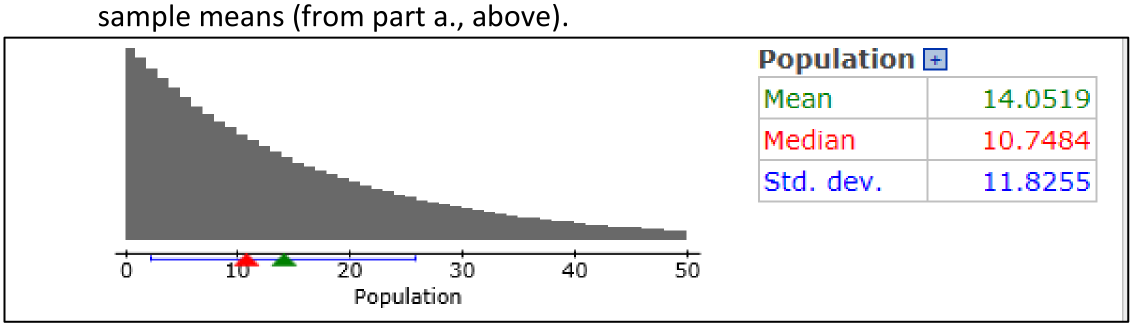


a. What does this histogram represent?

b. What is the basic shape of the histogram? ____

c. What is the mean? ____________________________

d. Let's compare the original uniform (below) with the corresponding sampling distributions of sample means (from part a., above).



1) How do the **basic shapes** compare?

2) How do the **means** compare?

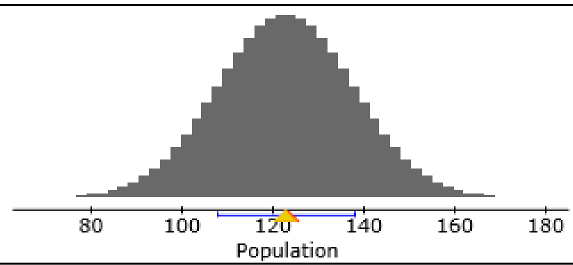
Part Four – Sampling Distributions from a Normal Population

1. Consider an original population that is approximately normal with a mean of 123 and a standard deviation of 15.
   a. What do you think the distribution of the original population looks like?
      Describe it and sketch it below.

   b. If you draw samples of size 2 from this same population and find the average (mean) of those 2 values, estimate what would a histogram of 1000 of those means look like.
      Describe it and sketch it below.

2. Compare your answers in the previous question to the simulated results below.

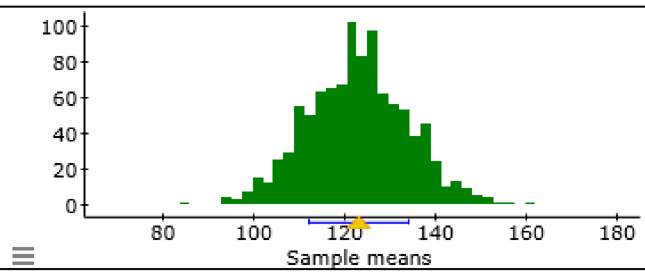
ORIGINAL POPULATION



Population

Mean	123
Median	123
Std. dev.	15

SAMPLE MEANS



Sample means

# of Samples	1000
Mean	123.1316
Median	123.1061
Std. dev.	10.882

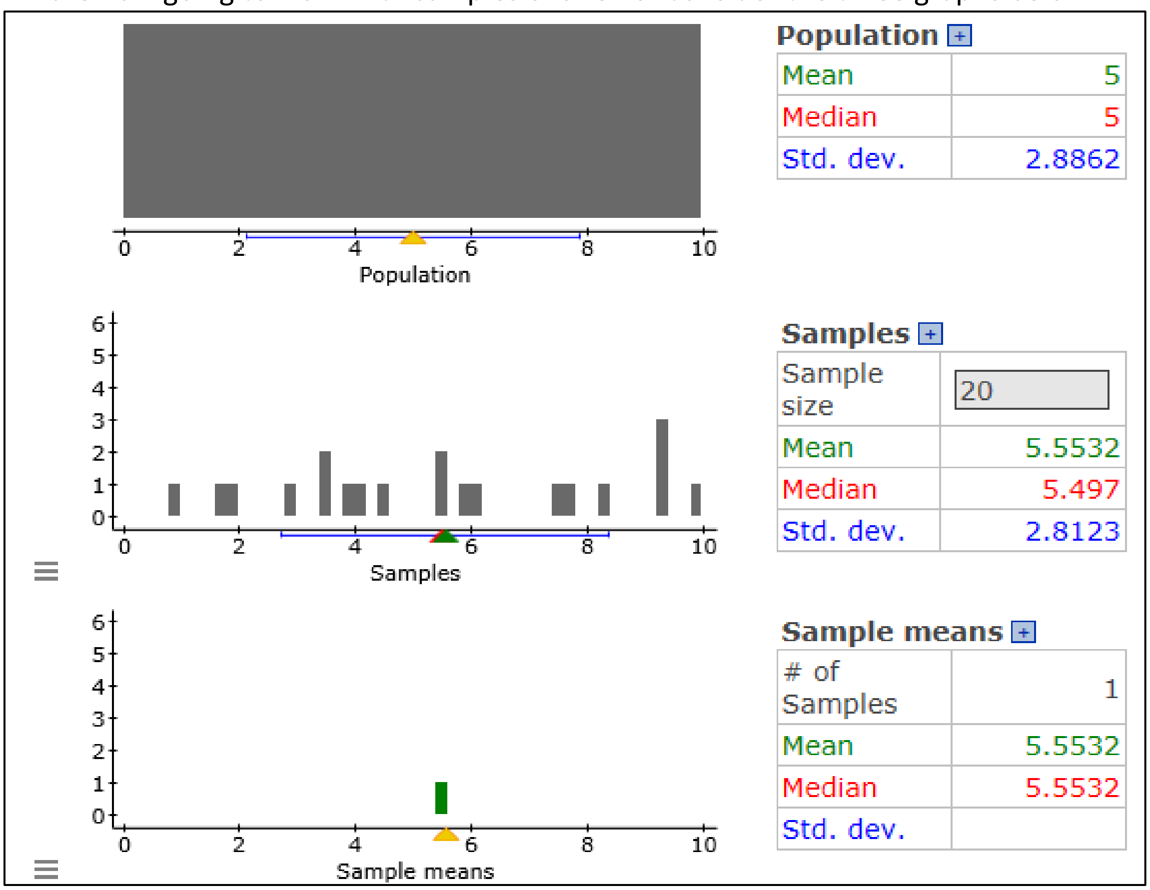
3. Refer back to the graphs in the previous question (number 2).
   a. Compare the **basic shapes** between the original population and the sample means.

   b. Compare the **means** between the original population and the sample means.

4. What general conclusions do you think you might be able to make (so far) about the mean (or average value) of the distribution of sample means?

Part Five – Increasing the Sample Size (*n* = 20)

1. Now let's focus on what happens if you increase the sample size. Instead of samples of size 2, we are now going to work with samples of size 20. Consider the three graphs below:

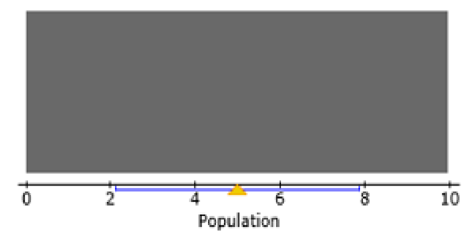
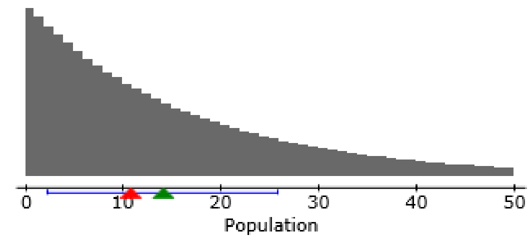
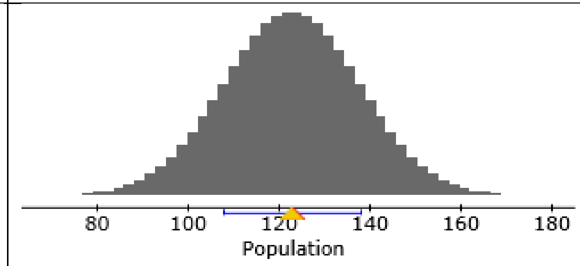
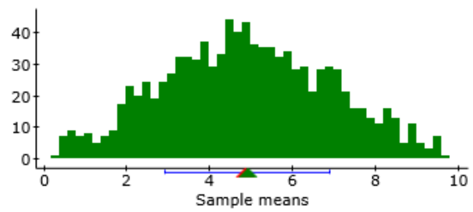
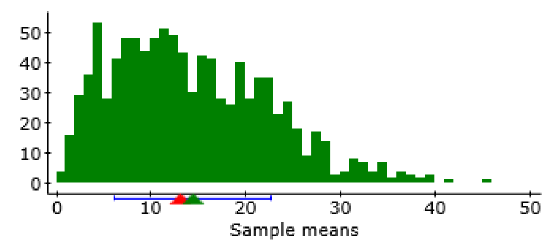
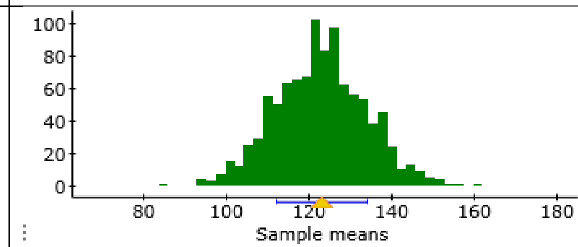
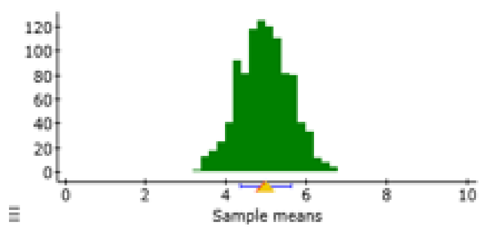
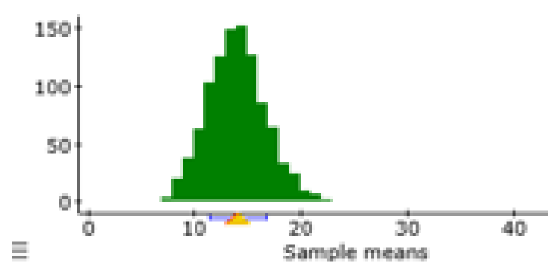
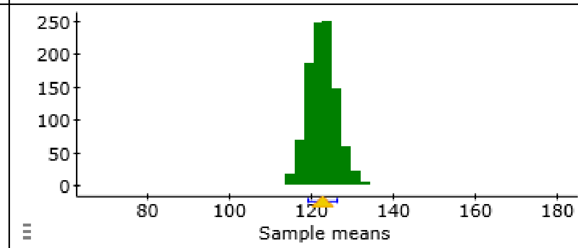


   a. **Top:** What is the population shape? ____

   b. **Middle:** What size sample is being taken from the population? ______________________

   c. **Bottom:** What is the mean of those 20 data points taken from that sample? ____________

2. For samples of size *n* = 20, compare the following graphs of original distributions with graphs 1000 sample means from each of the distributions.

	Uniform	**Skewed Right**	**Normal**
Popu-lation			
Sample *n* = 2			
Sample *n* = 20			

a. From the table, what can you conclude about the **means** of each distribution (population mean), compared with the means of sampling distributions?

b. For the larger sample sizes, what can you conclude about the **basic shapes** of the histograms?

c. Comparing sample sizes *n* = 2 and *n* = 20, what can you say about the spread (variability) of the sample mean distribution?

MAIN IDEAS: RECAP OF LEARNING OUTCOMES

You have just realized the Central Limit Theorem means. The details of it are given below.

CENTRAL LIMIT THEOREM:

The distribution of the sample mean ($\bar{x}$, or *x*-bar) has the following properties:

1. **Shape:** approximately normal (bell-shaped), if the sample size is sufficiently large ($n \geq 30$)
2. **Center (mean):** $\mu_{\bar{x}} = \mu$
3. **Spread (standard deviation):** $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

NOTE: The properties for mean and standard deviation are true, regardless of the sample size.

Now, try to state the ideas of the Central Limit Theorem (CLT) in your own words: